Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Numerical Methods

Subject Code: 4SC04MTE1/4SC04NUM1 Branch: B.Sc. (Mathematics, Physics)

Semester:4 Date:03/05/2018 Time:10:30 To 01:30 Marks: 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) Initial approximation of $x^3 - x - 2 = 0$ can be chosen from ___________. (01)

(1) $[0, 1]$ (2) $(-1, 0)$
(3) $(1, 2)$ (4) $[1, 2]$

b) For Simpson's $\frac{1}{3}^{rd}$ rule, n is required multiple of _____________. (01)

(1) 2 (2) 3
(3) 4 (4) 5

c) To derive Trapezoidal rule ____________ formula is used. (01)

(1) Newton's Backward (2) Gauss Forward
(3) Newton's Forward (4) Gauss Backward

d) Which of the following method can be used to evaluate a numerical integral (01)

(1) Picard's Method (2) Runge –Kutta Method
(3) Euler's Method (4) None of these

e) The order of convergence in Newton'sRaphson method is (01)

(1) 2 (2) 3
(3) 0 (4) None of these

f) **Match the following:** (01)

A	Newton-Raphson	1	Integration
B	Runge-kutta	2	Root finding
C	Simpson's Rule	3	Ordinary Differential Equations

(1) $A2 - B3 - C1$ (2) $A1 - B3 - C2$
(3) $A2 - B1 - C3$ (4) None of these

g) Newton's iterative formula to find the value of $\sqrt{N}$ is (01)

(1) $x_{n+1} = \left(x_n + \frac{N}{x_n}\right)$ (2) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$
(3) $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)$ (4) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{x_n}\right)$

h) Write Runge-Kutta second order method. (01)

i)	If $f(x)$ is given by	(01)

x	0	0.5	1
$f(x)$	1	0.8	0.5

Then using Trapezoidal rule, find $\int_0^1 f(x)\ dx$.

j) The number of strips required in Weddle's rule is ………… (01)

k) Find an interval containing an initial approximation $x^2 - 10x + 7 = 0$. (01)

l) Write formula of Euler's modified method. (01)

m) Newton-Raphson method is applicable to the solution of both algebraic and transcendental equations. Determine whether the statement is True or False. (01)

n) Predictor-corrector methods is a self-starting method. Determine whether the statement is True or False. (01)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Calculate the value of $\int_0^1 \frac{x}{1+x}\, dx$ correct up to three significant figures, taking six intervals by using (i) Trapezoidal rule, (ii) Simpson's $\frac{1}{3}^{rd}$ rule. (07)

b) The function $f(x)$ is tabulated below, for different values of x (07)

x	0	5	10	15	20
$f(x)$	1.5708	1.5738	1.5828	1.5981	1.6200

Compute first and second derivatives of $f(x)$ at $x = 0$ and $x = 20$.

Q-3 **Attempt all questions** **(14)**

a) Derive differentiation formulae based on Newton's forward interpolation formula. (07)

b) State and prove Euler-Maclaurin Sum Formula. (07)

Q-4 **Attempt all questions** **(14)**

a) Find a positive root of $x + \ln x - 2 = 0$ by Newton-Raphson method correct to two significant figure. (05)

b) Compute $y(2)$, if $y(x)$ satisfies the equation $\frac{dy}{dx} = \frac{1}{2}(x + y)$ given $y(0) = 2$, $y(0.5) = 2.636, y(1.0) = 3.595$ and $y(1.5) = 4.968$, using Milne's method. (05)

c) Find a root of the equation $x^x + 2x - 6 = 0$, by method of bisection, correct to two decimal places. (04)

Q-5 **Attempt all questions** **(14)**

a) Let $x = \xi$ be a root of $f(x) = 0$ and let I be an interval containing the point $x = \xi$. Let $\phi(x)$ and $\phi'(x)$ be continuous in I where $\phi(x)$ is defined by the equation $x = \phi(x)$ which is equivalent to $f(x) = 0$. Then prove that if $|\phi'(x)| < 1$ for all x in I, the sequence of approximations $x_0, x_1, x_2, \ldots, x_n$ defined by $x_n = \phi(x_{n-1})$ converges to the ξ, provided that the initial approximation x_0 is chosen in I. (05)

b) Use Taylor's series method to compute $y(1.1)$, correct to five decimal places, when $y(x)$ satisfies the equation $\frac{dy}{dx} = xy$ with $y(1.0) = 2$. (05)

c) Find $y(0.10)$ and $y(0.15)$ by Euler's method, from the differential equation $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 0$ correct up to four decimal places, taking step length $h = 0.5$. (04)

Q-6 **Attempt all questions** **(14)**

a) Using Regula-Falsi method, find a root of $x \sin x = 1$ correct to three decimal places. (05)

b) Derive differentiation formula based on Newton's divided difference formula. (05)

	c)	Evaluate: $\int_0^1 \frac{dx}{1+x^2}$ by using Weddle's rule with $h = \frac{1}{6}$.	(04)
Q-7		**Attempt all questions**	**(14)**
	a)	Evaluate $\int_{0.1}^{0.7}(e^x + 2x)dx$, by Simpson's $\frac{3}{8}^{th}$ rule, taking $h = 0.1$, correct up to five decimal places.	(05)
	b)	Compute $y(0.2)$, by Runge-Kutta fouth order method correct up to four decimal places, from the equation $\frac{dy}{dx} = x + y,\ y(0) = 1$, taking $h = 0.2$.	(05)
	c)	Describe Picard's Method for first order ordinary differential equation.	(04)
Q-8		**Attempt all questions**	**(14)**
	a)	Obtain Picard's second approximate solution of the initial value problem $\frac{dy}{dx} = \frac{x^2}{y^2+1}, y(0) = 0$.	(05)
	b)	Find the root of $x^2 + \ln x - 2 = 0$, which lies between 1 and 2 by iteration method correct up to four decimal places.	(05)
	c)	Apply Euler-Maclaurin sum formula to find the sum $1^3 + 2^3 + 3^3 + \cdots + n^3$.	(04)